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INCOMPLETE SPLIT-PLOT DESIGNS BASED ON α-DESIGNS

A COMPROMISE BETWEEN TRADITIONAL SPLIT-PLOT DESIGNS AND RANDOMISED COMPLETE BLOCK DESIGNS

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Introduction

- **Example of trial to be performed**
- 2-factorial design
  - Treatment factor 1 with few levels (e.g. ± Herbicides)
  - Treatment factor 2 with many levels (e.g. a large number of varieties)
- **Some possible designs**
- Split-plot
- Randomised complete block designs
- Incomplete split-plot

What is an incomplete split-plot

Small example: ±Herbicide, 9 varieties

Randomised complete block design

9	**5**	2	**8**	9	5	4	**6**	**4**	1	**1**	8	**7**	3	**2**	7	**3**	6
5	7	2	9	4	**4**	8	3	5	6	**7**	**1**	**6**	**2**	**8**	1	**3**	**9**

Traditional split-plot

2	9	5	8	1	4	7	6	3	**9**	**8**	**5**	**6**	**4**	**1**	**2**	**3**	**7**
7	**2**	**9**	**3**	**8**	**4**	**1**	**6**	**5**	7	1	6	5	2	8	9	3	4

Incomplete split-plot

8	2	5	**8**	**2**	**5**	1	4	7	**7**	**4**	**1**	**3**	**6**	**9**	9	6	3
9	**7**	**8**	7	8	9	**4**	**6**	**5**	6	5	4	2	1	3	**2**	**1**	**3**

Construction

- Can be based on different types of incomplete block designs
  - Chosed to use to use α-designs (generalised lattice)

- α-designs (generalized lattice)
  - Resolvable
  - Flexible
    - Available for almost any number of varieties ($v>2$)
    - Wide rage of replications ($2 \leq r \leq 10$)
    - Broad range of block sizes (and v does not need to be a multiple of k)

Construction – 4 different methods

v=12, r=2 (in A and B) or *4 (in C and D), s=4, k=3.*

Design method	Rep	Layout in the “field”							
A	1	D H L	C G K	c g k	b f j	a e i	A E I	B F J	d h l
	2	c h i	C H I	d e j	B G L	A F K	b g l	D E J	a f k
B	1	B F J	b f j	A E I	a e i	d l h	D L H	C G K	c g k
	2	d e j	D E J	C H I	c h i	a f k	A F K	B G L	b g l
C	1	D H L	C G K	c e l	b h k	a g j	A E I	B F J	d f i
	2	c f l	C H I	a h j	B G L	A F K	b e k	D E J	d g i
D	1	B F J	b h k	a g j	A E I	D H L	d f i	c e l	C G K
	2	d g i	D E J	C H I	c f l	A F K	a h j	B G L	b e k

Analyses

$$Y_{rtv} = \mu + \delta_v + \tau_t + (\delta\tau)_{vt} + A_r + E_{rtv}$$ RCB

$$Y_{rtv} = \mu + \delta_v + \tau_t + (\delta\tau)_{vt} + A_r + C_{rt} + E_{rtv}$$ Split-plot

A and C

$$Y_{rbtv} = \mu + \delta_v + \tau_t + (\delta\tau)_{vt} + A_r + D_{rb} + E_{rbtv}$$

$$Y_{rgbtv} = \mu + \delta_v + \tau_t + (\delta\tau)_{vt} + A_r + C_{rg} + D_{rgb} + E_{rgbtv}$$ B and D

Intra-block efficiencies

Trial definition				Efficiency of α-design used for construction[1]		Efficiency of variety comparisons		Efficiency of F-test for the hypothesis of no interaction	
Replicates r	Treatments t	Varieties v	Blocks s	Method A&B	Method C&D	Method A&B	Method C&D	Method A&B	Method C&D
2	2	80	10	0.806	0.864	0.806	0.806	0.886	0.848
2	2	40	5	0.823	0.876	0.823	0.823	0.897	0.863
2	2	48	6	0.817	0.872	0.817	0.817	0.894	0.858
2	2	48	8	0.753	0.826	0.753	0.752	0.851	0.801
3	2	35	5	0.845	0.872	0.845	0.844	0.882	0.843

1) Also efficiency factor for varieties if no interaction effect are included in the model

- If no interaction effect then method C and D are better than method A and B
- If test for interaction is important then method A and B are better than method C and D
- If variation between group of blocks are larger than between blocks then
  - Method B is better than method A and method D is better than method C

The projects, in which the designs were used, expected interaction between the two factors and were very interested in testing for interaction, so method B were chosen

Incomplete split-plot

- Practical compromise
  - Easier than RCB, more difficult than split-plot
  - May require more guard-area than split-plot, but less than RCB
- Efficiency
  - Herbicides: more whole plots, comparison within pair (group) of incomplete block and thus moderate distance between incomplete "whole-plots": More efficient than split-plot
  - Varieties: few plots within each incomplete "whole plot" and thus small distance between sub-plots: More efficient that RCB and split-plot

Performed experiments

Trial no	Number of				
	Repli-cates r	Treat-ments t	Varie-ties v	Blocks s	Plots per block k
A	2	2	48	6	8
B	2	2	48	8	6
C	2	2	48	6	8
D	3	2	35	5	7
E	3	2	8	2	4

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Field layout of trial E

Each plot is
1.5 m × 11.0 m
Each block is
12.0 m × 11.0 m

Replicate 1						Replicate 2					
45	**03**	42	34	17	**14**	38	**42**	36	**29**	**31**	16
20	**25**	08	27	30	**31**	08	**25**	24	**14**	**02**	23
35	**04**	46	32	29	**15**	45	**46**	17	**18**	**12**	44
18	**16**	12	19	43	**09**	22	**09**	10	**41**	**11**	01
24	**37**	47	05	02	**48**	40	**21**	27	**05**	**20**	48
21	**10**	06	41	28	**07**	28	**19**	03	**37**	**32**	47
33	**23**	13	38	22	**01**	04	**15**	33	**43**	**34**	26
40	**11**	39	44	26	**36**	07	**30**	13	**39**	**06**	35
35	16	**47**	**19**	**17**	14	**40**	25	**03**	37	34	**35**
24	37	**08**	**44**	**28**	31	**45**	15	**27**	29	20	**01**
18	10	**39**	**32**	**43**	15	**08**	09	**13**	39	12	**26**
20	04	**13**	**05**	**22**	09	**07**	19	**10**	14	32	**48**
40	25	**06**	**38**	**29**	01	**28**	21	**17**	41	11	**16**
45	23	**46**	**34**	**30**	36	**38**	42	**36**	18	06	**44**
33	03	**42**	**41**	**26**	48	**04**	46	**33**	43	02	**47**
21	11	**12**	**27**	**02**	07	**22**	30	**24**	05	31	**23**

Field layout of trial E

Each plot is
2.5 m × 12.5 m
Each block is
10.0 m × 12.5 m

Rep 3	Variety	7.0	1.0	3.0	6.0	3.1	6.1	7.1	1.1
	plotno	6	7	18	19	30	31	42	43
	Variety	8.1	2.1	4.1	5.1	5.0	2.0	8.0	4.0
	plotno	5	8	17	20	29	32	41	44
Rep 2	Variety	7.1	6.1	5.1	8.1	7.0	8.0	6.0	5.0
	plotno	4	9	16	21	28	33	40	45
	Variety	2.0	4.0	3.0	1.0	2.1	4.1	3.1	1.1
	plotno	3	10	15	22	27	34	39	46
Rep 1	Variety	4.0	1.0	7.0	5.0	4.1	1.1	7.1	5.1
	plotno	2	11	14	23	26	35	38	47
	Variety	3.0	8.0	6.0	2.0	8.1	6.1	3.1	2.1
	plotno	1	12	13	24	25	36	37	48

Efficiency of the designs, Yield

Trial no.	Design[1]: Comparison	Grain yield, $hkg\ ha^{-1}$ LSD Plan	Split-plot	RCB	Relative reduction, % Split plot	RCB
A	Variety:Treat	6.5	7.9	8.04	18	19
	Treat:Variety	6.7	10.1	8.04	34	17
	Treat	2.2	11.3	1.16	81	-90
	Variety	4.6	5.6	5.69	18	19
B	Variety:Treat	8.4	8.5	9.19	1	9
	Treat:Variety	8.4	16.9	9.19	50	9
	Treat	1.7	23.2	1.33	93	-28
	Variety	6.2	6.0	6.50	-3	5
C	Variety:Treat	4.9	7.9	8.92	38	45
	Treat:Variety	5.3	19.7	8.92	73	41
	Treat	2.4	27.0	1.29	91	-86
	Variety	3.5	5.6	6.31	37	45
D	Variety:Treat	2.0	2.8	2.86	29	30
	Treat:Variety	2.2	3.1	2.86	29	23
	Treat	1.0	1.7	0.48	41	-108
	Variety	1.4	2.0	2.02	30	31
E	Variety:Treat	5.8	5.7	7.4	-2	21
	Treat:Variety	5.9	7.6	7.4	23	21
	Treat	3.7	7.8	2.6	53	-40
	Variety	4.2	4.1	5.2	-4	19

Efficiency of the designs, %Mildew

		Powdery mildew severity, transformed %				
Trial no.		LSD			Relative reduction, %	
	Design[1]: Comparison	Plan	Split-plot	RCB	Split plot	RCB
A	Variety:Treat	0.609	0.625	0.625	3	3
	Treat:Variety	0.609	0.695	0.625	12	3
	Treat	0.086	0.577	0.090	85	4
	Variety	0.440	0.442	0.442	0	0
C	Variety:Treat	0.950	1.032	1.032	7	7
	Treat:Variety	0.970	1.149	1.032	16	6
	Treat	0.263	0.954	0.149	72	-77
	Variety	0.672	0.730	0.730	8	8
D	Variety:Treat	0.437	0.437	0.438	0	0
	Treat:Variety	0.437	0.455	0.438	4	0
	Treat	0.074	0.175	0.074	58	0
	Variety	0.309	0.309	0.310	0	0

Efficiency of the designs, other variables

Variable	Design: Comparison	LSD: Plan	LSD: Split-plot R	LSD: RCB	Relative reduction, %: Split-plot R	Relative reduction, %: RCB
Wheat June 19 g $0.25m^{-2}$	Variety:Herbicide	161	167	195	3	17
	Herbicide:Variety	174	210	195	17	11
	Herbicide	119	204	69	42	-73
	Variety	115	118	138	3	17
Log Weed June 19 Log g $0.25m^{-2}$	Variety:Herbicide	2.8	2.8	2.8	1	0
	Herbicide:Variety	2.8	2.9	2.8	4	0
	Herbicide	1.0	2.1	1.0	53	0
	Variety	2.0	2.0	2.0	1	0
Vegetation cover (%) June 5	Variety:Herbicide	.052	.055	.077	6	33
	Herbicide:Variety	.055	.057	.077	3	30
	Herbicide	.032	.041	.027	21	-17
	Variety	.036	.039	.055	6	33
LAI June 21	Variety:Herbicide	0.59	0.60	0.59	2	1
	Herbicide:Variety	0.59	0.62	0.59	4	0
	Herbicide	0.26	0.44	0.21	41	-24
	Variety	0.42	0.42	0.42	2	1
MTA June 21	Variety:Herbicide	5.7	5.8	5.8	0	2
	Herbicide:Variety	5.7	6.0	5.8	4	2
	Herbicide	2.0	4.3	2.1	53	2
	Variety	4.1	4.1	4.1	0	2
Log DIFN June 21	Variety:Herbicide	0.52	0.52	0.53	-0	1
	Herbicide:Variety	0.53	0.57	0.53	7	0
	Herbicide	0.26	0.45	0.19	43	-38
	Variety	0.37	0.37	0.38	-1	1

Discussion and conclusions

- **Efficiency**
- Compared to randomised complete block design
  - Incomplete split-plot were most often less efficient when comparing the main effect of treatments
    - Larger number of independent plots in RCB
  - Incomplete split-plot most often more efficient for other comparisons
    - Larger block size in RCB
- Compared to traditional split-plot
  - Incomplete split-plot were most often more efficient for all types of comparisons
    - Especially for comparing treatment means (many more degrees of freedom and smaller blocks)

Discussion and conclusions

- Increase in efficiency
  - In most cases larger for grain yield than for mildew
    - Probably because mildew is less sensible to soil variation
  - Small for trial E when comparing mean of varieties and varieties within treatment
    - Relative small reduction in block sizes
  - Small for trial B when comparing mean of varieties and varieties within treatment
    - Reason unknown

Discussion and conclusions

- Practical considerations
  - Treatment applications
    - Easier than randomised complete block design
    - More difficult than split-plot design
  - Guard areas
    - Less than for randomised complete block design
    - More than for split-plot design
  - Design and statistical analysis
    - More complex than both randomised complete block design and split-plot design
    - Appropriate software are available and with today's computer power this should not be a problem